A Leader-Follower Strategy for Implementing Demand Responses in Power Distribution Systems

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***Abstract*— In this paper, a new dynamic pricing scheme is proposed for demand response aggregators (DRAs) to maximize their profits. A Stackelberg game is established to model the interactions between a load serving entity (LSE) and DRAs, and a bi-level optimization formulation attained. In the upper-level problem, the energy procurement from the wholesale electricity market is determined for the LSE based on the electricity price forecast, while DRAs adjust their consumption patterns with respect to the dynamic pricing scheme. The proposed model is transformed into a mixed-integer, quadratically-constrained quadratic programming (MIQCQP) problem, and the corresponding Karush-Kuhn-Tucker (KKT) conditions are attained to represent the optimality of the lower-level problem. The complementary constraints in the KKT conditions are linearized using big-M reformulation. Finally, the effectiveness of the proposed approach is demonstrated by numerical examples.**

***Index Terms* — Stackelberg game, dynamic pricing scheme, bi-level programming, non-cooperative game**

NOMENCLATURE

A. Indices and Sets

$\mathcal{N}$	Set of nodes, $\mathcal{N} := \{0,1,...,N\}$
$\mathcal{D}$	Set of indices of DRs, $\mathcal{D} := \{1,...,D\}$
$\mathcal{T}$	Set of indices of time slots, $\mathcal{T} := \{1,...,T\}$
Ω_L	Set of indices of loads
Ω_{RES}	Set of indices of renewable generation sources

B. Parameters

$\bar{\lambda}_t$	Market electricity price forecast at time *t*
γ_t^R	Retail price for the fixed load at time *t*
γ_t^{RES}	Retail price of renewable energy at time *t*
$\bar{P}_{i,t}^{RES}$	Upper limit of the renewable energy power at bus *i* at time *t*
$\bar{P}^{ch}$ / $\bar{P}^{dch}$	Upper limit of charging/discharging power for Energy Storage System (ESS)
η^{ch} / η^{dch}	ESS charging/discharging efficiency
E^B	ESS capacity (MWh)
$\mathcal{P}_{\mathcal{T},i}^L$	The regular load profile at bus *i*.
$\mathcal{P}_{\mathcal{T}}^L$	The total regular load profile, i.e., $\mathcal{P}_{\mathcal{T}}^L \triangleq \sum_{i\in\mathcal{N}} \mathcal{P}_{\mathcal{T},i}^L$
$\bar{v}_i$ / $\underline{v}_i$	Upper/lower limit of nodal voltage magnitude at bus *i*.
ϕ	Capacity of the inverter
$\bar{P}_{d,t}^{DR}$ / $\underline{P}_{d,t}^{DR}$	Upper/lower limit of DRA *d* power at time *t*
E_d^{DR}	The minimal energy consumption for DRA *d*
$\tilde{\vartheta}$	Step size of price signals

C. Variables

γ_t^{DR}	Retail price for DRA at time *t*
ϑ_t	The price signal at time *t*
$\mathcal{P}_{\mathcal{T},d}^{DR}$	Scheduled load profile for DRA *d*.
P_t^0	Power procurement from the wholesale market at time *t*
$P_{i,t}^{RES}$	Power procurement from the renewable energy at bus *i* at time *t*
P_t^{ch} / P_t^{dch}	Charging/discharging power for ESS at time *t*
SOC_t	ESS state-of-charge at time *t*
ψ	Integer variables

D. Uncertainty Variables

λ_t	Electricity price in wholesale market at time *t*

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I. INTRODUCTION

In general, the retail prices that utility companies charge end-users do not reflect the actual wholesale prices at the time of consumption. Thus, end-users have no incentive to change their consuming behavior that prevents an efficient utilization of available generation capacity [1]. To address this issue, one of the early strategies is to adjust the end-user's consumption level according to the wholesale price, leading to the advent of Demand-Side Management (DSM). To implement DSM, there are mainly two ways, i.e. the energy efficiency improvement program and demand response (DR) program. The DR programs can be further divided into two broad categories: price-based and incentive-based ones. The incentive-based DR program mainly includes direct load control, and interruptible/curtailable service while the price-based DR includes real-time pricing (RTP) and critical peak pricing (CPP) which can be considered as an indirect load control technique.

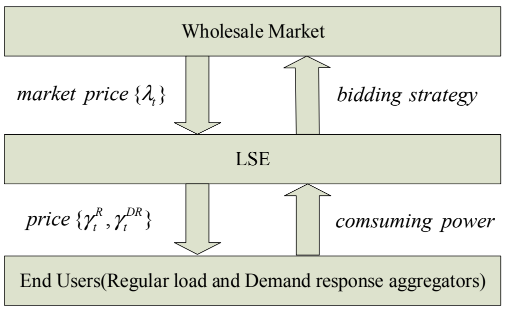


Fig. 1 The conceptual design of the proposed system model

Existing DR programs mainly focus on industrial and large commercial consumers [2]. However, with the deployment of a smart grid infrastructure and domestic controllable appliances, end-users can be involved in DR programs in the future. Many papers have studied price-based DR in the residential sector through aggregating controllable appliances and/or small customers. Usually, social welfare is maximized through coordinating energy provider generation and consumer demand under day-ahead pricing or real-time pricing (e.g. [3] - [5]). Other objective functions may also be used, such as minimizing power consumption [4] or network loss [6]. In [7] and [8], the authors proposed a bilevel model to determine a real-time dynamic pricing scheme to schedule DR resources.

In the three-level structure of the electricity market proposed in [8], as shown in Fig. 1, the LSE plays the role of an intermediary agent who buys electricity from the wholesale market and resells it to end-users. To address this issue, we focus on the price-based DR programs using the concept of real-time pricing and thereby propose a novel model to design the retail price for DRAs. Specifically, the LSE determines the pricing scheme to achieve social welfare maximization while the end-users (e.g. DRAs) optimize their power consumption based on the pricing scheme. In the view of game theory, the hierarchical optimization problem between the LSE and the DRAs can be modeled as a Stackelberg game where the LSE is the leader, and the DRAs are the followers.

In existing publications, DRAs are commonly modeled as independent price takers (e.g. [7] - [8]). The problem inherent in this model is the load synchronization, where a part of load can be shifted from a typical peak hour to a nonpeak hour however without significantly reducing the peak-to-average ratio [9]. Different from the existing models where the followers are considered as price takers and only the leader determines the retail price, the DRAs (followers) can influence the retail price here. Specifically, a novel form of retail price determination is proposed whereby DRAs must consider their own and the actions of other players on the retail price in the decision-making process. As shown in Fig. 2, a non-cooperative game is constructed to model the interactions among DRAs at the lower level of the bilevel problem.

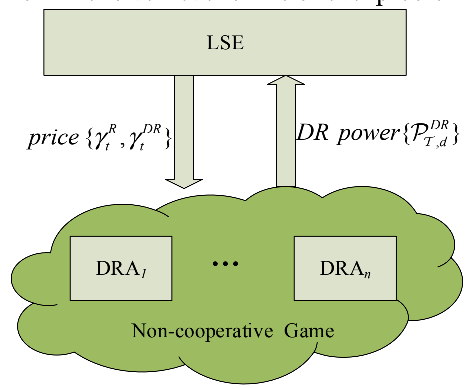


Fig. 2 Structure of the bilevel model

The contributions of this paper are threefold. First, power flow constraints and voltage limits are considered in the day-ahead dispatch strategy, and a more applicable model is attained for LSEs. Secondly, the LSE decides the optimal retail price for DRAs through anticipating the reactions of the DRAs in a Stackelberg game setting. Thirdly, a non-cooperative game model is formulated to simulate the interactions among the DRAs in the lower-level model, and the existence and uniqueness of a Nash equilibrium are analyzed. As the Nash equilibrium exists and is unique in our model, the KKT conditions are sufficient and necessary for the optimality of the lower level problem. Case studies show that our model can improve the operational efficiency of the distribution system.

The remainder of the paper is organized as follows: the dynamic pricing scheme is formulated as an optimization problem in Section II. The method to transform the model into a tractable MIQCQP model is given in Section III. The performance of the proposed model and method is demonstrated in Section IV. Conclusions together with future research prospect are presented in Section V.

II. PROBLEM FORMULATION

A. Notation

For the given vector $\mathbf{x}$, $\mathbf{y}$, $\mathbf{x} \perp \mathbf{y} \Leftrightarrow \mathbf{x}'\mathbf{y} = 0$ and $diag(\mathbf{x})$ denotes a diagonal matrix whose elements is composed of $\mathbf{x}$. $\mathbf{x}'$ denote the transpose of $\mathbf{x}$. For clarity, define the total scheduled load profile of DRA as $\mathcal{P}_{\mathcal{T},\mathcal{D}}^{DR} \triangleq \left[P_{\mathcal{T},1}^{DR}, \dots, P_{\mathcal{T},D}^{DR} \right]'$ and $\mathcal{P}_{\mathcal{D}}^{DR} \triangleq \sum_{d \in \mathcal{D}} \mathcal{P}_{\mathcal{T},d}^{DR}$. The notation $\mathcal{P}_{\mathcal{T},-d}^{DR}$ is used to denote the power consumptions of DRA other than DRA d, i.e., $\mathcal{P}_{\mathcal{T},-d}^{DR} := \left\{ \mathcal{P}_{\mathcal{T},1}^{DR}, \dots, \mathcal{P}_{\mathcal{T},d-1}^{DR}, \mathcal{P}_{\mathcal{T},d+1}^{DR}, \dots, \mathcal{P}_{\mathcal{T},D}^{DR} \right\}$.

B. The Upper Level Problem

Before the mathematical formulation is presented, the following assumptions are given:

Assumption 1: The LSE can use some mathematical tools to estimate the DR characteristics and further determine the corresponding model parameters for a DRA

Justification: Some papers such as [10] and [11] use the inverse optimization to model the demand-side consumption behavior via historical price-consumption data .

Assumption 2: There exists an agreement between the EUC and DRAs to specify an upper limit on the retail price. **Justification:** A large price signal would be unfair for DRAs and a similar assumption is made in [8].

It is assumed that the LSE is regulated so that its objective is not to maximize its profit through selling electricity, but rather to induce customers' consumption in a way that maximizes the social welfare [12], which is represented as follows:

Social Welfare= Utility-Cost

where

$$Utility = \sum_{d\in\mathcal{D}} \sum_{t\in\mathcal{T}} U_{d,t} \quad (1)$$

$$Cost = \sum_{t\in\mathcal{T}} \left(\lambda_t P_t^0 + \sum \gamma_t^{RES} \bar{P}_{i,t}^{RES}\right) \quad (2)$$

The constraints in the upper-level model include power flow equations, limits on nodal voltage magnitudes and hardware and operational constraints for the distributed energy resources along with the coupled inverter.

The power flow equations are linearized using the method presented in [13]. In fact, the nodal voltage magnitude is regulated into statutory ranges to maintain the power delivery, and is formulated as

$$\bar{v}_i \le \|V_i\|_2 \le \underline{v}_i, \forall i \in \mathcal{N} \quad (3)$$

Next, two representative DERs will be modeled including renewable energy source (RES) coupled with an inverter and an electric storage system (ESS).

For any RES $i \in \Omega_{RES}$, at any time slot $t \in \mathcal{T}$, the complex power S_i^{RES} must respect the following constraint:

$$S_i^{RES} \in \{(P_{i,t}^{RES}, Q_{i,t}^{RES}) \mid 0 \le P_{i,t}^{RES} \le \bar{P}_{i,t}^{RES}, Q_{i,t}^{RES} \le \sqrt{\phi_i^2 - (P_{i,t}^{RES})^2}\}. \quad (4)$$

Here it is assumed that only one ESS is integrated into the distribution network. The operational constraints take the following form:

$$\begin{cases} 0 \le P_t^{dch} \le \bar{P}^{dch}, 0 \le P_t^{ch} \le \bar{P}^{ch} \ \forall t \in \mathcal{T} \\ SOC_{t+1} = SOC_t + (P_t^{ch}\eta^{ch} / E^B - P_t^{dch} / E^B \eta^{dch}) \forall t \in \mathcal{T} \setminus \{T\} \\ SOC^{\min} \le SOC_t \le SOC^{\max} \quad \forall t \in \mathcal{T} \end{cases} \quad (5)$$

C. The Lower Level Problem

In the lower level, the objective is welfare maximization for each DRA. For any DRA $d \in \mathcal{D}$, it can be formulated as:

$$\underset{\mathcal{P}_{\mathcal{T},d}^{DR} \in \mathcal{X}_d^{DR}}{\text{maximize}} \sum_{t\in\mathcal{T}} U_{d,t} - \sum_{t\in\mathcal{T}} C_d^{DR}\left(P_{d,t}^{DR}\right) \quad (6)$$

where $\mathcal{X}_d^{DR}$ denotes the feasible region of the power consumption for DRA d and $C_d^{DR}(\bullet)$ denotes its cost function.

Here, a concave quadratic utility function is used, which corresponds to a linear decreasing marginal benefit. For any DRA $d \in \mathcal{D}$, at any time slot $t \in \mathcal{T}$, it takes the following form:

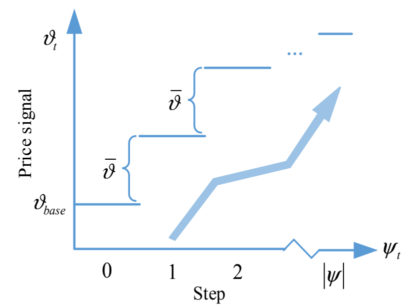


Fig. 3 The dynamic price signals ϑ_t

$$U_{d,t}\left(P_{d,t}^{DR}\right) = \begin{cases} \zeta_{d,t} P_{d,t}^{DR} - \upsilon_{d,t} (P_{d,t}^{DR})^2 & 0 \le P_{d,t}^{DR} \le \zeta_{d,t} / 2\upsilon_{d,t} \\ (\zeta_{d,t})^2 / 4\upsilon_{d,t} & P_{d,t}^{DR} \ge \zeta_{d,t} / 2\upsilon_{d,t} \end{cases} \quad (7)$$

where $U_d(P_{d,t}^{DR})$ is the utility function of the DRAs. For ease of computation, the quadratic utility function can be accurately approximated by an overestimating piecewise linear function composed by a set of tangent lines. Note that this approach is applicable even when the utility function is a multi-segment function. For any DRA $d \in \mathcal{D}$, at any time slot $t \in \mathcal{T}$, the utility function can be approximated as follows:

$$U_{d,t} \le a_{d,k,t} P_{d,t}^{DR} + b_{d,k,t} \qquad k = 1,\dots,K \quad (8)$$

where K is the number of segments. For any DRA $d \in \mathcal{D}$, at any time slot $t \in \mathcal{T}$, $\mathcal{X}_d^{DR}$ can be formulated as follows:

$$P_{d,t}^{DR} \le \bar{P}_{d,t}^{DR} \qquad \alpha_{d,t}^1 \ (9)$$

$$P_{d,t}^{DR} \ge \underline{P}_{d,t}^{DR} \qquad \alpha_{d,t}^2 \ (10)$$

$$U_{d,t}^{DR} \le u_{d,k,t} + IU_{d,k,t}(P_{d,t}^{DR} - P_{d,k,t}^{DR}) \ \forall k = 1,\dots,K \qquad \alpha_{d,k,t}^3 \ (11)$$

$$\sum_{t\in\mathcal{T}} P_{d,t}^{DR} \ge E_d^{DR} \qquad \alpha_d^4 \ (12)$$

where $\alpha_{d,t}^1, \alpha_{d,t}^2, \alpha_{d,k,t}^3, \alpha_d^4 \ge 0$ are the Lagrange variables.

To simplify (7), a linear price model is introduced to approximate $C_d^{DR}(\bullet)$. The payoff function for each DRA $\mathcal{Q}_d$ can be obtained:

$$\mathcal{Q}_d = \underset{\mathcal{P}_{\mathcal{T},d}^{DR} \in \mathcal{X}_d^{DR}}{\text{maximize}} \sum_{t\in\mathcal{T}} \left(U_{d,t} - \gamma_t^{DR} P_{d,t}^{DR}\right) \quad (13)$$

where

$$\gamma_t^{DR}\left(\mathcal{P}_{\mathcal{I},t}^{DR}, P_{\mathcal{I},t}^L\right) = \mathcal{K}\left(\mathcal{P}_{\mathcal{I},t}^{DR} + P_{\mathcal{I},t}^L\right) + \vartheta_t \quad \forall t \in \mathcal{T} \quad (14)$$

Usually, the parameters $\mathcal{K}$ and ϑ_t must be pre-determined [7]-[9]. Therefore, we assume that ϑ_t are variables and can be considered as the price signals to induce DRAs to adjust their consumptions. A ϑ_t can be set as a piecewise price function as shown in Fig. 3, corresponding to a multi-step price scheme which is widely applied in utility companies. Here, a step function for ϑ_t is employed, and stated as follows:

$$\vartheta_t = \vartheta_{base} + \tilde{\vartheta} * \psi_t \quad (15)$$

Hence, we have the following game among the DRAs:

Players: the DRAs in set $\mathcal{D}$

Strategies: each DRA $d \in \mathcal{D}$ chooses its power consumption $P_{d,t}^{DR} \in \mathcal{X}_d^{DR}$ to maximize its payoff function.

Payoff function: $\mathcal{Q}_d$ for each DRA $d \in \mathcal{D}$ is defined as in (13)

A Nash equilibrium of the game defined above is a vector $\mathcal{P}_{\mathcal{T},\mathcal{D}}^{DR(*)}$ such that for any $d \in \mathcal{D}$, we have

$$\mathcal{Q}_d\left(\mathcal{P}_{\mathcal{T},d}^{DR(*)},\mathcal{P}_{\mathcal{T},-d}^{DR(*)}\right)\geq \mathcal{Q}_d\left(\mathcal{P}_{\mathcal{T},d}^{DR},\mathcal{P}_{\mathcal{T},-d}^{DR}\right)\quad \mathcal{P}_{\mathcal{T},d}^{DR}\in\mathcal{X}_d^{DR} \tag{16}$$

Proposition 1: The Nash equilibrium of the game above always exists and is unique. The proof is given in Appendix A

The conditions of each player's optimal action can be described as a nonlinear complementarity problem (NCP), which will be detailed in Section III.

III. Solution Methodology

In this section, a robust formulation is proposed when the wholesale market price is considered as a random variable. Then the corresponding bilevel model is transformed into a single-level problem by solving the KKT conditions. At last, the complementarity constraints are linearized into disjunctive constraints to make the problem tractable.

A. The Robust Optimization Formulation

As the wholesale electricity price is random and unknown to operators until the day-ahead market is cleared, the LSE will try to minimize the total cost for purchasing electricity in the worst-case scenario. This problem can be solved using a robust optimization formulation. First, the uncertainty set for the random variable is defined in a polyhedral representation:

$$\Omega_\lambda=\{\lambda_t\mid\lambda_t=\bar{\lambda}_t+z_t\tilde{\lambda}_t,\forall t,\forall z_t\in\mathbf{Z}\} \tag{17}$$

where $\mathbf{Z}=\{z_t\,||\,z_t\,|\leq 1,\sum_{t\in\mathcal{T}}|\,z_t\,|\leq\Gamma,\forall t\in\mathcal{T}\}$. The constraint in (17) ensures that the values of λ_t stay in an interval. The set $\mathbf{Z}$ indicates that the total normalized deviations of the market price from its forecast throughout the scheduling horizon cannot exceed the predefined parameter Γ which is referred to as the *uncertainty budget*.

Thereby, the pertinent robust optimization problem can be described as follows:

$$\begin{cases} max\ \underset{\lambda_t}{min}\sum_{t\in\mathcal{T}}\sum_{d\in\mathcal{D}}\left(U_{d,t}-\lambda_t P_t^0-\sum\ \gamma_t^{RES}\bar{P}_{i,t}^{RES}\right)\\ s.t.\ P_t^0\in\Omega_0,\lambda_t\in\Omega_\lambda,\forall t\in\mathcal{T}\end{cases} \tag{18}$$

where Ω_0 represent the feasible region of P_t^0 .

As the min-max problem is still intractable for commercial solvers, the following equivalent form is introduced by utilizing the duality theory.

$$\max\sum_{t\in\mathcal{T}}\left(\sum_{d\in\mathcal{D}}U_{d,t}-\sum_{i\in\Omega_{RES}}\gamma_t^{RES}P_{i,t}^{RES}-\tilde{\lambda}_t P_t^0-\delta_t\right)-z_0\Gamma \tag{19}$$

$$s.t.\begin{cases}P_t^0\in\Omega_0\\ -\rho_t\leq P_t^0\leq\rho_t,\delta_t+z_0\geq\Delta\lambda_t\rho_t\quad\forall t\in\mathcal{T}\\ z_0\geq 0,\delta_t\geq 0,\rho_t\geq 0\end{cases} \tag{20}$$

where z_0 , δ_t are dual variables and ρ_t is the auxiliary variable.

B. Bilevel to One-level

The leader's problem in a Stackelberg game can be presented in the following compact form:

$$\min_x f\left(x,u_1,...,u_G\right) \tag{21}$$

$$s.t.\qquad (x,u_1,...,u_G)\in\mathcal{X} \tag{22}$$

$$u_g\in\begin{Bmatrix}\arg\min\limits_{y_i} h_i\left(x,u_1,...,u_G\right)\\ s.t.\,c_g\left(u_g\right)\leq 0\quad\alpha_g\end{Bmatrix}\forall g=1,...,G \tag{23}$$

If (23) is convex, the conditions that the followers choose the optimal strategy can be written through the KKT conditions as follows

$$\begin{cases}\nabla u_g h_g\left(x,u_1,...,u_G\right)-\nabla u_g c_g\left(u_g\right)\alpha_g=0\\ 0\leq\alpha_g\perp c_g\left(u_g\right)\leq 0\end{cases}\forall g=1,...,G \tag{24}$$

When the lower level problem (23) is a non-cooperative game and the Nash equilibrium uniquely exists, the model above can be viewed as an MPEC problem and can be transformed into a single-level problem as follows

$$\begin{cases}\min\limits_{x,u_1,...,u_G} f\left(x,u_1,...,u_G\right)\\ s.t.\{(22),(24)\}\end{cases} \tag{25}$$

For any DR $d\in\mathcal{D}$,any time slot $t\in\mathcal{T}$,the KKT conditions take the following form:

$$\mathcal{K}_t\left(P_{d,t}^{DR}+\mathcal{P}_{\mathcal{I},t}^{DR}+\mathcal{P}_{\mathcal{I},t}^{L}\right)+\vartheta_t+\alpha_{d,t}^1-\alpha_{d,t}^2-\sum_k a_{d,k,t}\alpha_{d,k,t}^3-\alpha_d^4=0 \tag{26}$$

$$\sum_{k=1}^K\alpha_{d,k,t}^3=1 \tag{27}$$

$$0\leq\bar{P}_{d,t}^{DR}-P_{d,t}^{DR}\perp\alpha_{d,t}^1\geq 0 \tag{28}$$

$$0\leq P_{d,t}^{DR}-\underline{P}_{d,t}^{DR}\perp\alpha_{d,t}^2\geq 0 \tag{29}$$

$$0\leq b_{d,k,t}+a_{d,k,t}P_{d,t}^{DR}-U_{d,t}^{DR}\perp\alpha_{d,k,t}^3\geq 0\ \ \forall k=1,...,K \tag{30}$$

$$0\leq -E_d^{DR}+\sum_{t\in\mathcal{T}}P_{d,t}^{DR}\perp\alpha_d^4\geq 0 \tag{31}$$

C. Linearization of Complementarity Constraints

Using the Big-M method, the complementary constraints (28)-(31) for each $d\in\mathcal{D}$ can be reformulated as the following linear constraints

$$\begin{cases}0\leq\bar{P}_{d,t}^{DR}-P_{d,t}^{DR}\leq M\beta_{d,t}^1\\ 0\leq\alpha_{d,t}^1\leq M\left(1-\beta_{d,t}^1\right)\end{cases} \tag{32}$$

$$\begin{cases}0\leq P_{d,t}^{DR}-\underline{P}_{d,t}^{DR}\leq M\beta_{d,t}^2\\ 0\leq\alpha_{d,t}^2\leq M\left(1-\beta_{d,t}^2\right)\end{cases} \tag{33}$$

$$\begin{cases}0\leq b_{d,k,t}+a_{d,k,t}P_{d,t}^{DR}-U_{d,t}^{DR}\perp\alpha_{d,k,t}^3\leq M\beta_{d,k,t}^3\\ 0\leq\alpha_{d,k,t}^3\leq M\left(1-\beta_{d,k,t}^3\right)\end{cases} \tag{34}$$

$$\begin{cases}0\leq\sum_{t\in\mathcal{T}}P_{d,t}^{DR}-E_d^{DR}\leq M\beta_d^4\\ 0\leq\alpha_d^4\leq M\left(1-\beta_d^4\right)\end{cases} \tag{35}$$

where M is a big number and β represents the binary variables.

Based on the methodology above, the original model is transformed into a MIQCQP problem, which is stated as:

$$\begin{cases}obj:(19)\\ s.t.\,(2)-(5),(20),(32)-(35)\end{cases} \tag{36}$$

Decision variables at the upper level include P_t^0 , P_t^{ch} , P_t^{dch} ϑ_t and S_i^{RES} while $P_{d,t}^{DR}$ is a decision variable at the lower level. Note that both ϑ_t and $P_{d,t}^{DR}$ determine the retail price for the DR resources.

IV. Numerical Results

In this section, case studies on a modified IEEE 123-bus test system are conducted to demonstrate the performance of our model. Numerical experiments are implemented in MATLAB on a personal computer with an Intel Core (i7, 2.80GHz) and

TABLE I
PARAMETER VALUE FOR ESS

$\bar{P}^{dch}$ (MW)	$\bar{P}^{ch}$ (MW)	E^{B} (MWh)	$SOC^{\max}$	$SOC^{\min}$	η^{ch}	η^{dch}
0.6	0.6	3	0.95	0.1	0.9	0.9

TABLE II
PARAMETER VALUES

Parameter	Value	Parameter	Value
γ^{R}	50\$/MWh	κ	38\$/MWh
γ^{RES}	30\$/MWh	ϑ_{base}	20\$/MWh
$\bar{P}^{DR}$	0.8MW	$\underline{P}^{DR}$	0.16MW
E^{DR}	11.6MWh	$\ddot{\vartheta}$	14 \$/MWh
υ	1100	ζ	630
Γ	12	$\tilde{\lambda}$	0.1 $\bar{\lambda}$
K	3		

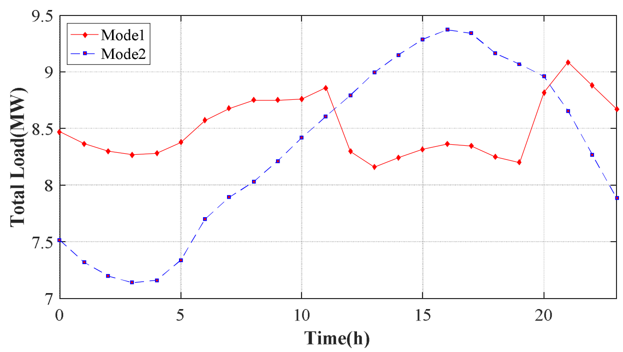


Fig. 4 Total power consumption in Modes 1 and 2

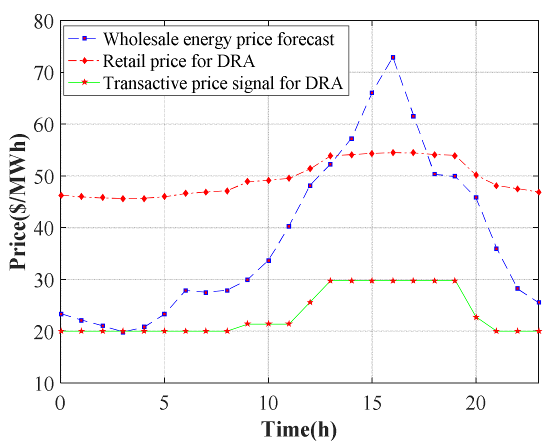


Fig. 5 Price for DRA in the 123-bus test system

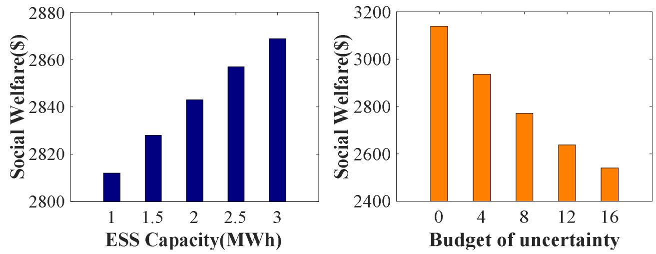
8GB memory. SCIP 3.2 [14] is invoked to solve the MIQCQP problem.

A. Simulation Data

Given that the PV is the dominant DGs in the medium (and low) voltage distribution system, only the PVs are considered in the test cases. The data for the two test systems and the solar irradiation data can be found in [15]. It is assumed all installed PVs (each coupled with an inverter with a capacity of 50kVA) share the same solar irradiance. Also, the ESS is owned by the LSE and integrated into the network at the slack bus. The parameters for the ESS are given in Table I.

TABLE III
COMPARISONS BETWEEN THE TWO MODES FOR 123-BUS SYSTEM

	Mode 1	Mode 2
Social welfare (\$)	2771.6	2374.9
DRA payment (\$)	4650.3	4830
DRA consumption (MWh)	98.87	96.66
Average price for DRAs (\$/MWh)	47.0	50.0
Network loss ratio (%)	6.70%	6.67%



(a) (b)

Fig. 6 Sensitivity analysis: (a) Impact of ESS capacity, (b) Impact of uncertainty budget

Both the electricity price forecasts, and the load variance are taken from the PJM website [16]. For simplicity, it is assumed all the DRAs share the same parameters. Note that 7 DRAs are connected at buses 13, 23, 44, 54, 72, 108, and 116 in the 123-bus system. The power factor of the DRAs is assumed to be 0.89. System parameters for the DRAs are given in Table II.

B. Performance Introduction

- **Mode 1**: In the proposed optimization model, the DRAs choose an optimal solution when receiving the price signals from the LSE.
- **Mode 2**: The regular retail price is applied to DRAs. In this mode, the DRAs have no incentive to modify their loads.

As the ESS is known to be able to reshape the load curve and has been widely applied in power systems, the impacts of ESS will be investigated in Section V-C. To better demonstrate the effectiveness of the presented model, in this section we assume that there is no ESS integrated into the network in mode 1 and mode 2. As shown in Fig. 4, our model can flatten the total load curve effectively. In combination with Fig. 5, when the electricity price in the wholesale market is high, e.g. between 10 am and 8 pm, the price signals increase and thus the power consumptions of the DRAs decrease. As in the time slot before 10 am, the price is relatively low, and thus the DRAs consumptions increase. Thus, the retail price for DRAs computed from our model can effectively reflect the underlying wholesale locational marginal price.

As we can see from Table III, the social welfare in mode 1 is remarkably higher than that in mode 2. The cost of purchasing electricity from LSE reduces by \$179.7 and moreover the average electricity price for DRA is lower than that in mode 2. Thus, our model is beneficial to both the LSE and DRAs when comparing with mode 2 and thus is more practical. Note that the network loss ratio in mode 1 is slightly higher than that in mode 2 because the DRAs consume more power in mode 1 and, therefore, result in a high loss ratio.

C. Sensitivity Analysis

In this section, the impacts of the capacity of the ESS and the uncertainty budget will be examined. Note that only the IEEE 123-bus test system is employed here. To investigate the impacts, E^B is changed from 1 to 3 MWh at a step size of 0.5 MWh while other parameters remain the same as those in mode 1 of Section V-A. As shown in Fig. 6(a), the profit of the LSE increases as the capacity of ESS increases. It is reasonable because a larger capacity means that the ESS can charge (discharge) more energy when the wholesale electricity price is low (high). Hence, the LSE can avoid the peak price and purchase electricity at a relatively low price.

The budget is changed from 0 to 16 at a step size of 4 while other parameters remain the same as those in mode 1 of Section V-A. As shown in Fig. 6(b), the profit decreases as the uncertainty budget increases. This is justified because a higher uncertainty budget makes it more likely for the market price to deviate from the forecast to go against the LSE. As a result, the cost for purchasing electricity from the wholesale market increases. In general, the choice of the budget reflects the risk aversion of the LSE and its confidence on the price forecast.

V. Conclusions and Future Research Prospect

In this paper, a dynamic retail-pricing scheme is proposed to reflect the underlying wholesale electricity price, distribution costs and scarcity rent arising from distribution capacity constraints. The retail pricing considers the end users' optimal adjustment on load consumption. The bilevel model is transformed into a single-level model by solving the corresponding KKT conditions to represent the optimality of lower level problem. As smart grid technologies increasingly provide end-users with convenient access to real-time price information, our proposed model can be applied to induce more efficient utilization of available generation capacity.

In the proposed model, each DRA operates subject to its own operational or hardware constraints only. It's necessary to consider the coupling relationship among DRAs (and between DRAs and LSEs), e.g. spatiotemporal coupling. We will extend the proposed game theoretic model to consider the coupling relations and will report the results in future publications.

Appendix A

Define $\sigma(\mathbf{x}) = \sum_{i}^{n} \varphi_i(\mathbf{x})$ and the pseudo gradient of $\sigma(\mathbf{x})$ as:

$$g(\mathbf{x}) = [\nabla_1 \varphi_1(x), ..., \nabla_n \varphi_n(x)]' \quad (1A)$$

Definition: the function $\sigma(\mathbf{x})$ is diagonally strictly concave for **x,** if for every $\mathbf{x}^1$, $\mathbf{x}^0$, it has

$$(\mathbf{x}^1 - \mathbf{x}^0)' g(\mathbf{x}^0) + (\mathbf{x}^0 - \mathbf{x}^1)' g(\mathbf{x}^1) > 0 \quad (2A)$$

Next, we show the function $\sum_{d \in \mathcal{D}} \mathcal{Q}_d \left(\mathcal{P}_1^{DR}, ..., \mathcal{P}_D^{DR} \right)$ is diagonally strictly concave. According to (1A), it yields

$$g\left(\mathcal{P}_{\mathcal{T},1}^{DR}, ..., \mathcal{P}_{\mathcal{T},D}^{DR}, U_{\mathcal{T},1}^{DR}, ..., U_{\mathcal{T},D}^{DR} \right) = [\nabla_1 \mathcal{Q}_1, ..., \nabla_D \mathcal{Q}_D,]' \quad (3A)$$

where

$$[\nabla_1 \mathcal{Q}_1, ..., \nabla_D \mathcal{Q}_D,]' = [-\mathcal{K}\left(\mathcal{P}_{\mathcal{D}}^{DR} + \mathcal{P}_{\mathcal{T},1}^{DR} + \mathcal{P}_{\mathcal{I}}^{L} \right) + \vartheta, ..., -\mathcal{K}\left(\mathcal{P}_{\mathcal{D}}^{DR} + \mathcal{P}_{\mathcal{T},D}^{DR} + \mathcal{P}_{\mathcal{I}}^{L} \right) + \vartheta, \underbrace{1, ..., 1}_{D}]' \quad (4A)$$

Define:

$$\mathbf{x}^1 = \left[\mathcal{P}_{\mathcal{T},1}^{DR(1)}, ..., \mathcal{P}_{\mathcal{T},D}^{DR(1)}, U_1^{DR(1)}, ..., U_D^{DR(1)} \right] \quad (5A)$$

$$\mathbf{x}^0 = \left[\mathcal{P}_{\mathcal{T},1}^{DR(0)}, ..., \mathcal{P}_{\mathcal{T},D}^{DR(0)}, U_1^{DR(0)}, ..., U_D^{DR(0)} \right] \quad (6A)$$

Substituting (5A) and (6A) into (2A), as $\mathcal{K} > 0$ and $\mathbf{x}^1 \neq \mathbf{x}^0$, it attains

$$\mathcal{K} \left\| \mathcal{P}_{\mathcal{D}}^{DR(1)} - \mathcal{P}_{\mathcal{D}}^{DR(0)} \right\|_2^2 + \mathcal{K} \sum_{d \in \mathcal{D}} \left\| \mathcal{P}_{\mathcal{T},d}^{DR(1)} - \mathcal{P}_{\mathcal{T},d}^{DR(0)} \right\|_2^2 > 0 \quad (7A)$$

Thus, the function $\sigma(\mathcal{P}_{\mathcal{T},1}^{DR}, ..., \mathcal{P}_{\mathcal{T},D}^{DR})$ is diagonally strictly concave. In addition, $\mathcal{Q}_d\left(\mathcal{P}_{\mathcal{T},d}^{DR}, \mathcal{P}_{\mathcal{T},-d}^{DR} \right)$ is a concave function. According to Theorem 1 and Theorem 2 in [17], the Nash equilibrium always exists and is unique.

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