

ORIGINAL RESEARCH

Communication-resilient and convergence-fast peer-to-peer energy trading scheme in a fully decentralized framework

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Abstract

The wide deployment of distributed energy resources, combined with a more proactive demand-side management, is boosting the emergence of the peer-to-peer market. In the present study, an innovative peer-to-peer energy trading model is introduced, enabling a group of price-setting prosumers to engage in direct negotiations via a straightforward best-response approach. A Nash equilibrium problem (NEP) is initially formulated and a sufficient condition for the unique solution of the NEP is derived. Afterwards, an asynchronous and convergence-fast solving method is employed to determine the trading quantity and price. The efficiency and resilience of the presented method are demonstrated through a comprehensive case study.

KEYWORDS

energy economics, energy markets, peer-to-peer (P2P) energy trading, prosumer, nash equilibrium, communication resilience, convergence

1 | INTRODUCTION

Peer-to-peer (P2P) energy trading facilitates direct communication and energy exchange among prosumers outspread in distribution systems. These prosumers represent various stakeholders engaged in negotiations for energy trading quantities and prices independently, proactively, and anonymously, while preserving privacy.

The most direct approach toward addressing the aforementioned requirements is employing distributed optimization techniques, such as the Lagrangian relaxation-based method (LR-M), to solve an assignment problem within a pool-like local energy market [1] or a game-theoretic framework [2]. However, this approach is primarily limited by lack of economic intuition during the iteration process. For example, in LR-M [1], a prosumer often acts as a price taker, aiming to maximize the decomposed augmented Lagrangian function instead of their individual objective functions. Consequently, pricing is determined using gradient-like methods instead of a pricing model. In addition, certain limitations of LR-M, such as slow convergence (as revealed by Ullah and Park [1]) and

parameter tuning, render it unsuitable for real-world scenarios. Some existing publications focused on pairing peers for trading through models such as bilateral contracts [3] or continuous double auctions [4]. However, in these frameworks, the decision variables, i.e. the quantities of traded electricity and corresponding prices, must be discretized within the price- or quantity-matching algorithm. The existence of bilinear terms resulting from the multiplication of price and quantity, which could potentially attenuate the efficiency of the outcomes, is the reason underlying the aforementioned requirement. In terms of communication resilience, the price adjustment process [3] and iterative double auction [4] can implement asynchronous transactions, where each agent takes one match in every time period. However, both processes are limited by suboptimality owing to the discretization issue mentioned above and exhibit an upper bounded gap in social welfare maximization. LR-Ms [1, 2, 5] require operation in the synchronous mode and are thus vulnerable to communication failures [5].

To address these limitations, a communication-resilient and convergence-fast P2P energy-trading scheme is proposed in this study. The major contributions of this work are as follows.

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- 1) A sufficient condition for the unique solution of the **Nash equilibrium problem (NEP)** is provided with a clear explanation of its economic rationale using the cobweb theory in economics.
- 2) A straightforward best-response algorithm, such as those outlined by Pandzic et al. [6] and Wang [7], is designed, and aligns with the negotiation mechanism in the P2P energy market to derive the solution.
- 3) The convergence analysis of the best-response algorithm is validated using a proposed iterative clearance mechanism for facilitating implementation in practical scenarios.

The remainder of this paper is organized as follows. In Section 2, the basic P2P energy-trading scheme is elaborated. Section 3 presents the solution methodology and certain validations. Numerical results are presented in Section 4 to validate the proposed models and algorithms. Finally, Section 5 presents the conclusions drawn and the scope for future research.

2 | PROPOSED P2P ENERGY TRADING SCHEME

A local energy trading market with I prosumers, who can either be buyers or sellers at any given moment, is considered. Sellers are presumed to offer quantities, whereas buyers bid on trading prices according to a negotiation mechanism. Consequently, a stable trading coalition constituting multiple sellers and one buyer will be formed because sellers always choose the highest bidder, and buyers do not exhibit any preference for energy from distinct sellers because energy is a homogeneous product. Therefore, the prosumers naturally form several coalitions in each of which the I prosumers, denoted by index i , can be divided into $(I - 1)$ sellers and one buyer, denoted by $i = I$. An iterative clearance mechanism is proposed in Section 3 to address the scenario where multiple sellers and buyers exist to attain a stable market outcome. Subsequently, a gaming model for P2P energy trading is established.

2.1 | Sellers' model

The sellers must minimize their operational costs according to the buyer's bidding price. For seller i at time t , the operational problem can be modelled as follows:

$$\max_{\text{PS}_{i,t}} C_{i,t}(\text{PS}_{i,t}) - \pi_t \times \text{PS}_{i,t} \quad (1)$$

$$C_{i,t}(\text{PS}_{i,t}) = 0.5 \times a_{i,t} (\text{PS}_{i,t})^2 + b_{i,t} \text{PS}_{i,t} + c_{i,t} \quad (2)$$

$$\text{PS}_{i,t}^{\min} \leq \text{PS}_{i,t} \leq \text{PS}_{i,t}^{\max} \quad (3)$$

where $a_{i,t}$, $b_{i,t}$, and $c_{i,t}$ are the parameters of the seller's cost function. π_t is the bidding price from the buyer at time t . $\text{PS}_{i,t}$ is the selling power of seller i at time t . $\text{PS}_{i,t}^{\min}$ and $\text{PS}_{i,t}^{\max}$ are the minimum and maximum generation outputs of the seller.

2.2 | Buyer's model

The buyer must establish a pricing model that considers the price elasticity of sellers, which can be estimated based on historical market outcomes. For seller i at time t , the supply curve can be approximated as a linear function:

$$\text{PB}_{i,t} = \frac{\pi_t - \tilde{b}_{i,t}}{\tilde{a}_{i,t}} \quad (4)$$

where $\tilde{b}_{i,t}$ and $\tilde{a}_{i,t}$ are parameters in the estimated supply curve and obviously $\tilde{a}_{i,t} \approx a_{i,t}$ which follows readily from the second-order derivative (1). $\text{PB}_{i,t}$ represents the estimated buying power from seller i at time t . Therefore, the pricing model can be expressed as follows:

$$\min_{\pi_t} \pi_t \times \sum_i \text{PS}_{i,t} + \gamma \sum_i d_i \text{PB}_{i,t} - U_t \left(\sum_i \text{PB}_{i,t} \right) \quad (5)$$

$$U_t \left(\sum_i \text{PB}_{i,t} \right) = \begin{cases} \lambda_t \sum_i \text{PB}_{i,t} - 0.5\beta_t \left(\sum_i \text{PB}_{i,t} \right)^2 \sum_i \text{PB}_{i,t} \\ \leq \lambda_t / \beta_t \\ \lambda_t^2 / (2\beta_t) \text{ otherwise} \end{cases} \quad (6)$$

$$\pi_t^{\min} \leq \pi_t \leq \pi_t^{\max} \quad (7)$$

$$\text{PB}_t^{\min} \leq \sum_i \text{PB}_{i,t} \leq \text{PB}_t^{\max} \quad (8)$$

where γ is network usage charge per unit electrical distance, and d_i is power transfer distance between seller i and the buyer. π_t^{\min} and π_t^{\max} are the feed-in tariff (FiT) and retail price, respectively. The utility function for buyer U_t is a piece-wise concave quadratic function, and λ_t and β_t are the parameters of the buyer's utility function, which can be determined by the buyer. PB_t^{\min} and PB_t^{\max} are the minimum and maximum power demanded by the buyer, respectively. Note that time index t is omitted hereafter except when stated otherwise.

Remark: An appropriate assumption is that the P2P trading price should be higher than the FiT but lower than the retail price, as shown in (7), which could effectively guarantee benefits to both buyers and sellers.

2.3 | Game model

Based on decision-making models of the seller and buyer, P2P energy trading can be formulated as a gaming model.

- 1) Players: sellers and buyers denoted by $i \in \mathcal{I} \triangleq \{1, \dots, I\}$.
- 2) The strategy set for player i , denoted by \mathcal{Q}_i , that is, (2)–(3) for a seller or (7)–(8) for a buyer, is closed and convex.
- 3) The disutility function for player i , denoted by f_i , that is, (1) for a seller or (5) for a buyer, is continuous over set \mathcal{Q}_i .

$\hat{\mathbf{f}} = (f_i)_{i=1}^I$, and the joint strategy set is obviously a Cartesian product of each player's set, i.e. $\mathcal{Q} = \prod_{i=1}^I \mathcal{Q}_i$. Therefore, the gaming model is formally expressed as tuple $\mathcal{G} = \langle \mathcal{Q}, \mathbf{f} \rangle$. For brevity, we denote the decision variables of player i as \mathbf{x}_i . $\mathbf{x} \triangleq (\mathbf{x}_1, \dots, \mathbf{x}_I)$ denotes the vector of all players' decision variables, while $\mathbf{x}_{-i} \triangleq (\mathbf{x}_1, \dots, \mathbf{x}_{i-1}, \mathbf{x}_{i+1}, \dots, \mathbf{x}_I)$ denotes the vector of all players' decision variables except that of player i . Therefore, the optimization problem for the buyers and sellers can be expressed in a compact form.

$$\begin{cases} \min_{\mathbf{x}_i} f_i(\mathbf{x}_i, \mathbf{x}_{-i}) \\ s.t. \mathbf{x}_i \in \mathcal{Q}_i \end{cases} \quad (9)$$

3 | SOLUTION METHODOLOGY

3.1 | Existence and uniqueness in the variational inequality VI formulation

For facilitating the analysis of the existence of the Nash equilibrium and convergence, the game model could be reformulated as a VI problem. Given that strategy set \mathcal{Q}_i is convex and closed and its (dis)utility function is continuously differentiable in \mathbf{x}_i for every fixed \mathbf{x}_{-i} , the game \mathcal{G} is equivalent to a VI problem (denoted by $\text{VI}(\mathcal{Q}, \mathbf{F})$) of finding a feasible vector $\mathbf{x}^* \in \mathcal{Q}$ such that [8, Prop. 1.4.2]

$$(\mathbf{x} - \mathbf{x}^*) \mathbf{F}(\mathbf{x}^*) \geq 0 \quad \forall \mathbf{x} \in \mathcal{Q} \quad (10)$$

where \mathbf{F} is the vector-valued function and $\mathbf{F}(\mathbf{x}) \triangleq (\nabla_{\mathbf{x}_i} f_i(\mathbf{x}))_{i=1}^I$.

Building on the existence/uniqueness results for the VI, the following lemma can be obtained:

Lemma 1. [9]: *Given the $\text{VI}(\mathcal{Q}, \mathbf{F})$, suppose that \mathcal{Q} is closed and convex, and \mathbf{F} is continuous on \mathcal{Q} . If \mathbf{F} is strongly monotone on \mathcal{Q} , the game \mathcal{G} has a unique Nash equilibrium (NE).*

Verifying these properties via the direct application of the definition is not infeasible. To streamline the subsequent analysis of the unique solution and the convergence of the proposed algorithm, two real matrices \mathbf{Y}_F and $\mathbf{\Gamma}_F$ are introduced, as expressed as:

$$\begin{aligned} [\mathbf{Y}_F]_{ij} &\triangleq \begin{cases} \alpha_i^{\min} & \text{if } i = j \\ -\theta_{ij}^{\max} & \text{otherwise} \end{cases} \quad \text{and } [\mathbf{\Gamma}_F]_{ij} \\ &\triangleq \begin{cases} 0 & \text{if } i = j \\ \theta_{ij}^{\max} / \alpha_i^{\min} & \text{otherwise} \end{cases} \end{aligned} \quad (11)$$

with

$$\alpha_i^{\min} \triangleq \inf_{\mathbf{x} \in \mathcal{Q}} \lambda_s(\boldsymbol{\varphi}_i^T \mathbf{J}_i \mathbf{F}_i(\mathbf{x}) \boldsymbol{\varphi}_i) \quad \text{and}$$

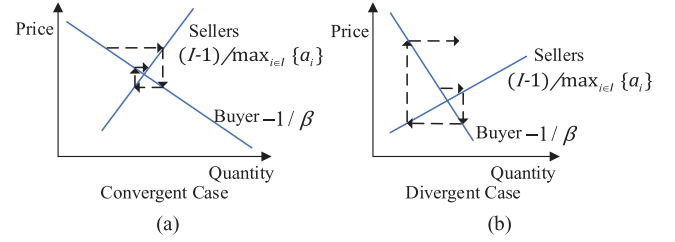


FIGURE 1 Economic interpretation for Lemma 2:(a) the convergent case; (b) the divergent case.

$$\theta_{ij}^{\max} \triangleq \sup_{\mathbf{x} \in \mathcal{Q}} \|\boldsymbol{\varphi}_i^T \mathbf{J}_j \mathbf{F}_i(\mathbf{x}) \boldsymbol{\varphi}_j\| \quad (12)$$

where $\boldsymbol{\varphi}_i$ with $i = 1, \dots, I$, is a set of arbitrary non-singular square matrices. $\lambda_s(\mathbf{A})$ denotes the least eigenvalue of \mathbf{A} . $\mathbf{J}_j \mathbf{F}_i(\mathbf{x})$ is the Jacobian of $\mathbf{F}_i(\mathbf{x})$,

Based on the above definitions, the P-matrix is invoked to study the existence and uniqueness of $\text{VI}(\mathcal{Q}, \mathbf{F})$, which also plays a crucial role in the convergence of the proposed algorithm.

Definition 1. Matrix \mathbf{M} is called P-matrix if every principal minor in \mathbf{M} is positive.

Remark: Although the related matrices in the presented model are real, we still introduce the properties of the complex matrix in the VI theory to study the model for consistency and rigor. According to the properties of the P-matrices, establishing the relation between the monotony of \mathbf{F} and P-property of \mathbf{Y}_F , as shown subsequently, is easy.

Proposition 1. [8]: *Given that \mathbf{F} is continuously differentiable with bounded derivatives on the closed and convex set \mathcal{Q} , if \mathbf{Y}_F is a P-matrix, then \mathbf{F} is strongly monotone on \mathcal{Q} .*

Combined with Lemma 1 and proposition 1, the sufficient condition for the game $\mathcal{G} = \langle \mathcal{Q}, \mathbf{F} \rangle$ to have a unique solution is that matrix \mathbf{Y}_F is a P-matrix.

Lemma 2. *Matrix \mathbf{Y}_F for the game model \mathcal{G} is a P-matrix when $\beta > \max_{i \in I} \{a_i\} / (I - 1)$.*

Proof. See Appendix.

Next, we explain the economic rationale behind Lemma 2 using the cobweb model shown in Figure 1. The condition in Lemma 2, that is, $\beta > \max_{i \in I} \{a_i\} / (I - 1)$, is equivalent to $(I - 1) / \max_{i \in I} \{a_i\} > 1/\beta$. The left side of the latter inequality can be regarded as the slope of the aggregated supply curve of the $(I - 1)$ sellers while $-1/\beta$ is the slope of the buyer's demand curve. When the demand curve is more elastic than the supply curve, i.e. $(I - 1) / \max_{i \in I} \{a_i\} > 1/\beta$, the fluctuation in each iteration is successively closer to the intersection model attains a unique solution. In addition, when β is large, i.e. $1/\beta$ is small, the demand curve is relatively elastic; therefore, the optimal

response algorithm that will be detailed in Section 3.2 can reach equilibrium relatively easily. While in the divergent case $\frac{I-1}{\max_{i \in I} \{a_i\}} < 1/\beta$, the fluctuations increase with each cycle and thus it cannot reach the equilibrium as in the right diagram.

Remark. The condition in Lemma 2 provides the lower bound for β . Such a condition is naturally satisfied in practical scenarios because the sellers with the renewable energy usually have near zero marginal cost and further the term on the left side goes to zero when $I \rightarrow \infty$.

3.2 | Solution algorithm and convergence analysis

Preliminary definitions should be provided to formally describe the asynchronous algorithm. Let $\mathcal{N}_i \subseteq \mathcal{N} = \{0, 1, 2, \dots\}$ denote the set of times at which \mathbf{x}_i is updated and $\tau_j^i(n)$ denote the most recent time at which player i receives the message from player j at the n -th iteration, and obviously satisfies $0 \leq \tau_j^i(n) \leq n$. In addition, we conduct the convergence analysis of the proposed best-response algorithm based on the partial asynchronism [8]. Specifically, communication delays are bounded. A positive integer B is assumed to exist, such that

$$n - B \leq \tau_j^i(n) \leq n \quad (13)$$

This implies that each player can perform an update at least once during any time interval of length B . B is commonly known as an asynchronism measure that represents the longest possible communication delay. The asynchronous algorithm based on the players' best responses is described as follows.

Asynchronous Best-Response Algorithm (ABRA)

(S.0) Choose any feasible $\mathbf{x}^{(0)}$ and set $n = 0$.

(S.1) Check the termination criterion.

(S.2) For each player i , compute (9).

(S.3) Set $n = n + 1$ and go to S.1.

Lemma 3. *If \mathbf{Y}_F is a P-matrix or equivalently $\rho(\mathbf{\Gamma}_F) < 1$, any sequence generated by ABRA converges to the NE of \mathcal{G} .*

Proof. According to [9, Theorem 10], ABRA is a block contraction. Then, based on the theorem in [10, Prop. 2.1], every limit point of ABRA is a fixed point of $\text{argmin } \mathbf{f}$, namely, the NE of \mathcal{G} . As proved by Lemma 3, the synchronous version of ABRA is a contraction with a modulus $\|\mathbf{\Gamma}_F\| \leq 1$, meaning that it would converge to the NE geometrically with the rate $\|\mathbf{\Gamma}_F\|$. Therefore, it easily determines the number of iterations required to achieve the desired accuracy σ .

Lemma 4. *Denote the NE of \mathcal{G} by \mathbf{x}^E . If $\|\mathbf{x}^{(n)} - \mathbf{x}^E\| \leq \sigma$ for any positive $n \geq \bar{n}$, then $\bar{n} = \log\left(\frac{\sigma(1-\|\mathbf{\Gamma}_F\|)}{\|\mathbf{x}^{(1)} - \mathbf{x}^{(0)}\|}\right) / \log(\|\mathbf{\Gamma}_F\|)$.*

Proof. See Appendix.

ABRA shares the same convergence performance as the synchronous version. ABRA shares the same convergence properties as its synchronous version. However, unlike the synchronous version, the bound on the convergence rate of ABRA is given by asynchronism measure B . For further details, please refer to [10, Prop. 3.2].

3.3 | Implementation mechanism

The P2P market simultaneously constitutes multiple buyers and sellers to negotiate the traded quantity and price. An iterative clearance mechanism is proposed to make our model (i.e. a one-buyer-multiple-seller game model) compatible with this practical scenario. In the first round of the clearance mechanism, each buyer forms a one-buyer-multi-seller game model, which consists of one buyer and all sellers in the market. Then, all these game models are solved, and the buyer who offered the lowest price at equilibrium would obviously have the right of first refusal to purchase electricity. Then, the remaining buyers and sellers simply iteratively repeat the same thing as in the past round until the sellers, buyers, or both are insufficient.

In addition, in each round of the clearance mechanism, the buyers or sellers only solve their decision-making model, that is, to computer (9), without declaring any private information to the central operator or other participants. Information exchanges among participants consist only of traded quantities and prices.

4 | CASE STUDY

In this case study, 10 prosumers (9 sellers and 1 buyer) were simulated, and the cost (or utility) parameters were cited from the study by Chen et al. [2]. Numerous gradient-like methods [11], including the basic projection method (BPM), Tikhonov regularization scheme (TRS), and proximal point scheme (PPS), for solving monotone VIs exist. In the present study, ABRA was compared with three other methods to demonstrate its superiority. As shown in Figure 2(a), the best-response-based scheme converges in only three iterations, whereas the gradient-like algorithms require 200 or more iterations to achieve a comparable performance. Clearly, the proposed method has a higher efficiency for market clearing, which is crucial for the practical engineering implementation of P2P transactions.

In addition, the resilience of the solution method was verified when confronting communication failures. ϵ is the probability of packet dropout. If a packet dropout occurs, the player performs a local update using the most recent information. As shown in Figure 2(b), the ABRA only needs 7 iterations to reach

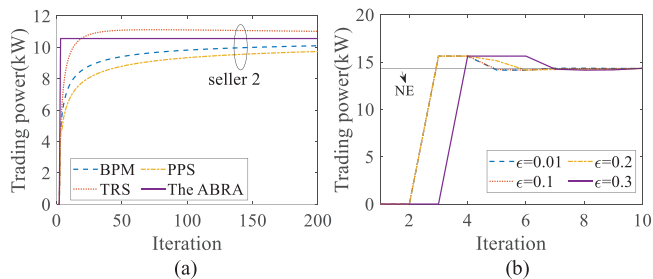


FIGURE 2 Numerical results: (a) comparison of the iteration process for the four methods; (b) robust performance with different probabilities of packet dropout.

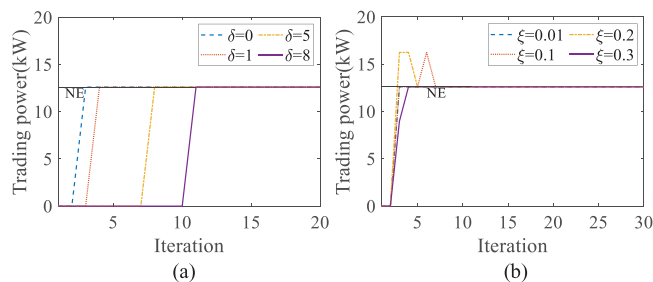


FIGURE 3 Robust performance: (a) with different time delays; (b) with different levels of data errors.

the NE even though ϵ increases to 0.3, which is sufficiently large in practical engineering scenarios.

To validate its practicality, we tested our method in the following two scenarios: delayed input and data errors, which are common in practical engineering cases. δ denotes the number of delayed iterations. Specifically, the player would have to perform the local update with the information in the past δ -th iteration. As shown in Figure 3(a), the ABRA monotonically reaches the NE even though δ increases to 8. In addition, Gaussian noise with the probability of ξ is presumably added to the information exchanged among the players to simulate the data errors. The mean value of the noise was set to 10% of the truth value. As shown in Figure 3(b), the ABRA could still reach the NE even though the probability ξ increases to 0.3, which is sufficiently large in practical scenarios.

Meanwhile, the total revenue of the proposed game model shares the same value as the social optimum; therefore, the price of anarchy (PoA) is 1, which shows that the proposed method achieves the same efficiency as the centralized method that maximizes social welfare.

5 | CONCLUSION AND FUTURE WORK

A communication-resilient and convergence-fast P2P energy trading scheme was established based on the NEP model. A sufficient condition was developed for the unique solution of the NEP, and insights into its economic underpinnings provided using the cobweb model. To derive the solution, an

asynchronous best-response algorithm was employed, and a subsequent analysis of its convergence was presented. The case study results demonstrate the high convergence speed and exceptional resilience of ABRA. In particular, the NEP model exhibited no loss of efficiency. More advanced ARBA-enabled market designs which can be robust against the parameters in the player's model should be studied in the future.

CONFLICT OF INTEREST STATEMENT

The authors declare no conflicts of interest.

DATA AVAILABILITY STATEMENT

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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REFERENCES

- Ullah, M.H., Park, J.D.: Peer-to-peer local energy trading with voltage management under asynchronous communication. *IEEE Trans. Smart Grid* 13(6), 4969–4972 (2022)
- Chen, Y., Zhao, C., Low, S.H., et al.: An energy sharing mechanism considering network constraints and market power limitation. *IEEE Trans. Smart Grid* 14(2), 1027–1041 (2023)
- Morstyn, T., Teytelboym, A., Mcculloch, M.D.: Bilateral contract networks for peer-to-peer energy trading. *IEEE Trans. Smart Grid* 10(2), 2026–2035 (2019)
- Haggi, H., Sun, W.: Multi-round double auction-enabled peer-to-peer energy exchange in active distribution networks. *IEEE Trans. Smart Grid* 12(5), 4403–4414 (2021)
- Guo, Z., Pinson, P., Wu, Q., et al.: An asynchronous online negotiation mechanism for real-time peer-to-peer electricity markets. *IEEE Trans. Power Syst.* 37(3), 1868–1880 (2022)
- Pandzic, H., Conejo, A.J., Kuzle, I.: An EPEC approach to the yearly maintenance scheduling of generating units. *IEEE Trans. Power Syst.* 28(2), 922–930 (2013)
- Wang, C., Wei, W., Wang, J., Liu, F., Mei, S.: Strategic offering and equilibrium in coupled gas and electricity markets. *IEEE Trans. Power Syst.* 33(1), 290–306 (2018)
- Facchinei, F., Pang, J.S.: *Finite-Dimensional Variational Inequalities and Complementarity Problem*. Springer, New York, NY (2003)
- Scutari, G., Facchinei, F., Pang, J.S., Palomar, D.P.: Real and complex monotone communication games. *IEEE Trans. Inf. Theory* 60(7), 4197–4231 (2014)
- Bertsekas, D.P., Tsitsiklis, J.N.: *Parallel and Distributed Computation: Numerical Methods*. Prentice-Hall, Upper Saddle River, NJ (1989)
- Kannan, A., Shanbhag, U.V.: Distributed iterative regularization algorithms for monotone Nash games. In: *Proceedings of IEEE Conference on Decision and Control (CDC)*, pp. 1963–1968. IEEE, Piscataway, NJ (2010)

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APPENDIX

Proof of Lemma 2

Without loss of generality, we set $\varphi_i = \mathbf{I}$ for each player; \mathbf{I} is an identity matrix. Obviously, \mathbf{Y}_F is the comparison matrix of the Hessian matrix of \mathbf{F} . \mathbf{Y}_F as a P-matrix is equivalent to \mathbf{Y}_F as a non-singular M-matrix. Therefore, the sufficient condition for Lemma 2 can be determined by generalizing the notion of strict diagonal dominance, which is stated as follows: the matrix \mathbf{Y}_F is a P-matrix if an entry-wise positive vector $\mathbf{w} = (w_i)_{i=1}^I$ exists, such that,

$$\frac{1}{w_i} \sum_{j \neq i} w_j \frac{\theta_{ij}^{\max}}{\alpha_i^{\min}} < 1, \forall i = 1, \dots, I \quad (14)$$

Specifically, it can be rewritten as follows:

$$w_I < a_I w_i, \forall i = 1, \dots, I-1 \text{ and } \sum_{j \neq I} w_j < w_I \beta \left(\sum_{i=1}^{I-1} 1/\tilde{a}_i \right)^2 \quad (15)$$

Next, we must prove whether \mathbf{w} exists to satisfy the inequalities in Equation (13). We further assume that $a_1 w_1 = \dots = a_{I-1} w_{I-1}$ and we only need to prove $\beta \left(\sum_{i=1}^{I-1} 1/\tilde{a}_i \right)^2 / \sum_{i=1}^{I-1} w_i > (I-1) / \sum_{i=1}^{I-1} a_i w_i$. Based on the fact that $\tilde{a}_i \approx a_i$ and $\beta > \max_{i \in I} \{a_i\} / (I-1)$, we have

$$\begin{aligned} & \beta \left(\sum_{i=1}^{I-1} 1/\tilde{a}_i \right)^2 / \sum_{i=1}^{I-1} w_i \\ & > \max_{i \in I} \{a_i\} \times \left(\sum_{i=1}^{I-1} 1/\tilde{a}_i \right)^2 / (I-1) \times \sum_{i=1}^{I-1} w_i \end{aligned}$$

$$\begin{aligned} & > \max_{i \in I} \{a_i\} \times \left[(I-1) \min_{i \in I} \{1/\tilde{a}_i\} \right]^2 / (I-1) \times \sum_{i=1}^{I-1} w_i \\ & \approx (I-1) / \max_{i \in I} \{a_i\} \times \sum_{i=1}^{I-1} w_i > (I-1) / \sum_{i=1}^{I-1} a_i w_i \quad (16) \end{aligned}$$

which provides conclusive evidence. In addition, a real symmetrical matrix is positive definite only if it belongs to P-matrices. Therefore, Lemma 2 can also be proved by deriving the critical conditions, making \mathbf{Y}_F positive definite.

Proof of Lemma 4

$\mathbf{x}^{(0)} \in \mathcal{Q}$ is fixed and the sequence $\{\mathbf{x}^{(n)}\}$ generated by the best response algorithm $\mathbf{x}^{(n)} = \mathbf{\Gamma}_F \mathbf{x}^{(n-1)}$ considered. According to the properties of block contraction, we have

$$\|\mathbf{x}^{(n+1)} - \mathbf{x}^{(n)}\| \leq \|\mathbf{\Gamma}_F\| \|\mathbf{x}^{(n)} - \mathbf{x}^{(n-1)}\| \quad (17)$$

for all $n \geq 1$, which implies

$$\|\mathbf{x}^{(n+1)} - \mathbf{x}^{(n)}\| \leq \|\mathbf{\Gamma}_F\|^n \|\mathbf{x}^{(1)} - \mathbf{x}^{(0)}\| \quad (18)$$

It follows that for each $n \geq 1$ and $m \geq 1$, we have

$$\begin{aligned} \|\mathbf{x}^{(n+m)} - \mathbf{x}^{(n)}\| & \leq \sum_{j=1}^m \|\mathbf{x}^{(n+j)} - \mathbf{x}^{(n+j-1)}\| \\ & \leq \|\mathbf{\Gamma}_F\|^n \left(1 + \|\mathbf{\Gamma}_F\| + \|\mathbf{\Gamma}_F\|^2 + \dots + \|\mathbf{\Gamma}_F\|^{m-1} \right) \|\mathbf{x}^{(1)} - \mathbf{x}^{(0)}\| \\ & \leq \frac{\|\mathbf{\Gamma}_F\|^n}{1 - \|\mathbf{\Gamma}_F\|} \|\mathbf{x}^{(1)} - \mathbf{x}^{(0)}\| \quad (19) \end{aligned}$$

Therefore, $\{\mathbf{x}^{(n)}\}$ is a Cauchy sequence that converges to a certain limit. Define the $\mathbf{x}^{(n+m)}$ as the NE of \mathbf{G} , and the desired result is obtained.